

Semester One Examination, 2022
Question/Answer booklet

**MATHEMATICS
SPECIALIST
UNIT 1**

If required by your examination administrator, please place your student identification label in this box

**Section One:
Calculator-free**

WA student number: In figures

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In words

Your name

Time allowed for this section

Reading time before commencing work: five minutes
Working time: fifty minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	7	7	50	50	35
Section Two: Calculator-assumed	12	12	100	92	65
Total					100

Instructions to candidates

1. The rules for the conduct of Trinity College examinations are detailed in the *Instructions to Candidates* distributed to students prior to the examinations. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.
3. You must be careful to confine your answer to the specific question asked and to follow any instructions that are specified to a particular question.
4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
5. It is recommended that you do not use pencil, except in diagrams.
6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section One: Calculator-free

35% (50 Marks)

This section has **seven** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

Question 1

(6 marks)

C_1 and C_2 are chords in the same circle. Let P be the statement 'chords C_1 and C_2 have equal lengths' and Q be the statement 'chords C_1 and C_2 subtend equal angles at the centre'.

- (a) Write, in words, the negation of P . (1 mark)
- (b) Write, in words, the meaning of $P \Rightarrow Q$. (1 mark)
- (c) Write, symbolically, the converse of $P \Rightarrow Q$. (1 mark)
- (d) Write, in any form, the inverse of $P \Rightarrow Q$. (1 mark)
- (e) Write, in any form, the contrapositive of $P \Rightarrow Q$ and state whether the contrapositive is true or false. (2 marks)

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Question 2

(5 marks)

Use the inclusion-exclusion principle to determine how many integers between 1 and 131 inclusive are divisible by 2, 3 or 8.

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See next page

Question 3

(7 marks)

Two vectors are $\mathbf{a} = 6\mathbf{i} - 8\mathbf{j}$ and $\mathbf{b} = \mathbf{i} - 3\mathbf{j}$.

(a) Determine the vector projection of \mathbf{a} on \mathbf{b} .

(3 marks)

(b) Vector $\mathbf{c} = x\mathbf{i} + y\mathbf{j}$ has twice the magnitude of \mathbf{a} and is perpendicular to \mathbf{b} . Determine the values of the constants x and y . (4 marks)

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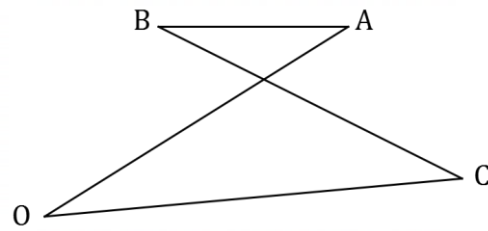
Question 4

(8 marks)

Crossed quadrilateral $OABC$ is shown in the diagram.

The midpoints of sides OA, AB, BC and CO are P, Q, R and S respectively.

Let $\vec{OA} = \mathbf{a}$, $\vec{OB} = \mathbf{b}$ and $\vec{OC} = \mathbf{c}$.



(a) Draw quadrilateral $PQRS$ on the diagram. (1 mark)

(b) Determine expressions for \vec{OP} , \vec{OS} , \vec{OQ} and \vec{OR} in terms of \mathbf{a} , \mathbf{b} and \mathbf{c} . (3 marks)

(c) Prove that the midpoints of the sides of a crossed quadrilateral join to form a parallelogram. (4 marks)

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Question 5

(8 marks)

Let $\mathbf{a} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$ and $\mathbf{c} = \begin{pmatrix} 4 \\ -5 \end{pmatrix}$.

(a) Determine

(i) $4\mathbf{a} + 2\mathbf{c}$.

(2 marks)

(ii) $|\mathbf{c} - \mathbf{b}|$.

(2 marks)

(b) Given that $\mathbf{a} = \lambda\mathbf{b} + \mu\mathbf{c}$, determine the value of the constant λ and the value of the constant μ .

(4 marks)

Question 6

(8 marks)

- (a) ABC is an isosceles triangle in which $BA = BC$. If M is the midpoint of AC , use a vector method to show that $\overrightarrow{BA} + \overrightarrow{BC} = 2\overrightarrow{BM}$. (3 marks)

- (b) $ABCD$ is a parallelogram and M is the midpoint of DC . Diagonal DB intersects AM at Q so that $\overrightarrow{AQ} = h\overrightarrow{AM}$ and $\overrightarrow{DQ} = k\overrightarrow{DB}$. Use a vector method to determine the value of the constant h and the value of the constant k . (5 marks)

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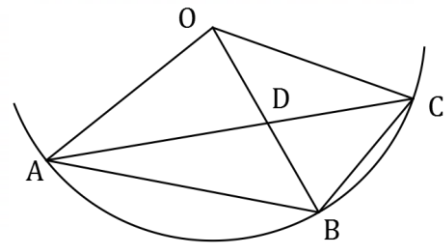
Question 7

(8 marks)

- (a) Points A, B and C lie on an arc of a circle with centre O as shown at right.

Chord AC intersects OB at point D .

The diagram is not drawn to scale.

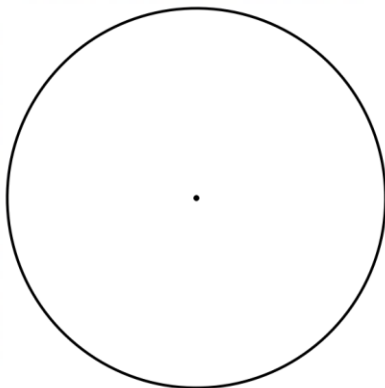


When $\angle ABC = 122^\circ$ and $\angle BCA = 33^\circ$, determine the size of $\angle BDC$.

(4 marks)

- (b) A secant cuts a circle with centre O at points M and N . Secant MN is extended beyond N to point P , where it meets a line that is a tangent to the circle at point Q . Prove that the size of $\angle NPQ$ is equal to one half the difference of the sizes of $\angle MOQ$ and $\angle NOQ$.

(4 marks)



Supplementary page

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